APPLYING THE AMBISONICS APPROACH ON PLANAR AND LINEAR ARRAYS OF LOUDSPEAKERS

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ABSTRACT

Near-field compensated higher order Ambisonics (HOA) is an approach to the physical synthesis of sound fields. The typical interpretation of the modern HOA approach is that the sound field to be synthesized and the spatio-temporal transfer function of the employed loudspeakers are expanded into series of spherical harmonics in order to determine the loudspeaker driving signals for spherical arrays. Going one level higher in abstraction, HOA can be interpreted as the single-layer potential solution to the problem which is solved via a formulation of the reproduction equation in a spatial frequency domain which is selected according to the geometry of the loudspeaker array. For spherical arrays, this spatial frequency domain is the spherical harmonics domain. The presented paper provides an overview over a recent extension of this approach to the employment of planar and linear loudspeaker arrays. In this latter situation, the solution is obtain via a formulation of the reproduction equation in wavenumber domain.

1. INTRODUCTION

Near-field compensated higher order Ambisonics (HOA) and wave field synthesis (WFS) constitute the two best known representatives of analytical methods for sound field synthesis. WFS directly implements fundamental physical principles like the Rayleigh integrals or the Kirchhoff-Helmholtz integral [1,2] and thus the possibilities and limitations of WFS can be deduced in an abstract manner from the physical fundamentals.

HOA on the other hand was developed from rather intuitive yet physical considerations [3]. The theory was later extended [4] and recently a solid physical interpretation in terms of the *single-layer potential* solution was found [5] which retroactively justifies the approach.

High-resolution HOA formulations such as [4–6] provide solutions only for spherical and circular arrays of loudspeakers. The operator theory which is applied in the single-layer potential solution enables the employment of arbitrarily shaped, simply enclosing loudspeaker contours [7] which greatly extends the flexibility of the HOA approach. This paper presents an extension of the Ambisonics approach to planar and linear loudspeaker contours which constitute special cases in the theory of single-layer potentials.

For convenience, we use the term *Ambisonics* in the remainder in order to refer to HOA. As discussed in Sec. 6, the notion of discrete *orders* is not suitable in the context of this paper.

2. AMBISONICS WITH SPHERICAL SECONDARY SOURCE DISTRIBUTIONS

This section reviews modern formulations of Ambisonics such as [5,6] for illustration purposes. At first stage, these approaches assume a continuous spherical distribution of secondary sources. This continuous distribution is then discretized in order to find the loudspeaker driving signals. The discretization operation is not treated in this paper. We will therefore refer to *secondary sources* rather than to loudspeakers in order to emphasize the continuous property.

The *reproduction equation* formulated for a spherical secondary source contour of radius R centered around the coordinate origin is given by [6]

$$S(\mathbf{x},\omega) = \int_0^{2\pi}\int_0^{\pi} D\left(\mathbf{x}_0,\omega\right)\; G\left(\mathbf{x}-\mathbf{x}_0,\omega\right)\; \sin\beta\; d\beta\; d\alpha\;, \quad (1)$$

whereby $D(\mathbf{x}_0,\omega)$ denotes the driving signal and $G(\mathbf{x}-\mathbf{x}_0,\omega)$ the spatio-temporal transfer function of the secondary source located at $\mathbf{x}_0$ with $\mathbf{x}_0 = R\;\left[\cos\alpha_0\sin\beta_0\;\; \sin\alpha_0\sin\beta_0\;\; \cos\beta_0\right]^T$. α_0 and β_0 denote the azimuth and colatitude of the secondary source position. $S(\mathbf{x},\omega)$ denotes the synthesized sound field. In order that (1) is valid $G(\mathbf{x}-\mathbf{x}_0,\omega)$ has to be invariant with respect to rotation around the coordinate origin [6].

The objective is to find the appropriate driving signals $D(\mathbf{x}_0,\omega)$ such that a given desired sound field $S(\mathbf{x},\omega)$ is properly synthesized.

Eq. (1) can be interpreted as a convolution along the surface of a sphere so that the convolution theorem

$$\mathring{S}_n^m(r,\omega) = 2\pi R\sqrt{\frac{4\pi}{2n+1}}\;\mathring{D}_n^m(\omega)\cdot\mathring{G}_n^0(r,\omega)\;, \quad (2)$$

employing the spherical harmonics expansion coefficients of $S(\mathbf{x},\omega)$, $D(\mathbf{x},\omega)$, and $G(\mathbf{x}-\mathbf{x}_0|_{r=R},\omega)$ respectively applies [8].

In the interior domain, a sound field $S(\mathbf{x},\omega)$ can be represented by the spherical harmonics coefficients $\mathring{S}_n^m(r,\omega)$ or coefficients $\breve{S}_n^m(\omega)$ respectively as

$$S(\mathbf{x},\omega) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n}\underbrace{\breve{S}_n^m(\omega)\; j_n\left(\frac{\omega}{c}r\right)}_{=\,\mathring{S}_n^m(r,\omega)}\; Y_n^m(\beta,\alpha)\;, \quad (3)$$

with $Y_n^m(\beta, \alpha)$ denoting the n-th order m-th degree spherical harmonic [9].

After reordering (2), it can be shown that [6]

$$\mathring{D}_n^m(\omega) = \frac{1}{2\pi R}\sqrt{\frac{2n+1}{4\pi}}\;\frac{\breve{S}_n^m(\omega)}{\breve{G}_n^0(\omega)} \;. \tag{4}$$

The secondary source driving function $D(\alpha, \beta, \omega)$ for three-dimensional reproduction of a desired sound field with expansion coefficients $\breve{S}_n^m(\omega)$ is then

$$D(\alpha, \beta, \omega) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} \underbrace{\frac{1}{2\pi R}\sqrt{\frac{2n+1}{4\pi}}\,\frac{\breve{S}_n^m(\omega)}{\breve{G}_n^0(\omega)}}_{=\,\mathring{D}_n^m(\omega)}\, Y_n^m(\beta, \alpha) \;. \tag{5}$$

Typically, the secondary sources are assumed to be monopole sources, e.g. [10, 11]. In principle, any secondary source transfer function that does not exhibit zeros in the spherical harmonics domain can be handled in the presented approach [12]. In practical applications the summation in (5) can not be performed over an infinite number of addends but is truncated at a specific value $N-1$. One speaks then of $(N-1)$-th order reproduction.

Note that above discussed approach corresponds to what is referred to as *near-field compensated higher order Ambisonics* [4]. A formulation of this approach employing circular distributions of secondary sources with a three-dimensional spatio-temporal transfer function - so-called *2.5-dimensional reproduction* - can be found in [6].

3. EXTENSION OF THE AMBISONICS APPROACH TO PLANAR SECONDARY SOURCE DISTRIBUTIONS

Further analysis of the Ambisonics approach as presented in Sec. 2 shows that it essentially constitutes the *single-layer potential* solution to sound field synthesis employing spherical distributions of secondary sources [5]. Eq. (1) can be understood as a compact Fredholm operator of zero index [7, 11, 13]. The general solution is obtained by expanding the operator and the virtual sound field into a series of orthogonal basis functions and performing a comparison of coefficients (i.e. *mode-matching*). For the spherical secondary source distribution treated in Sec. 2, these orthogonal basis functions are given by the spherical harmonics $Y_n^m(\beta, \alpha)$ and the convolution theorem (2) constitutes a mode-matching operation.

The Fredholm operator theory does not require the secondary source distribution to be spherical [13, 14]. Any contour simply enclosing the receiver volume is possible. Although the solutions to such more complicated contours are mathematically well understood, the required basis functions are only available for simple geometries like prolate spheroids and similar.

In order to find the solution to the reproduction equation for planar contours (i.e. in order to find the Ambisonics solution), we assume a boundary which consists of a disc Ω_0 and a hemisphere Ω_{hemi} of radius r_{hemi} as depicted in Fig. 1 [9]. As we let $r_{\text{hemi}} \to \infty$, the disc Ω_0 becomes an infinite plane and the volume under consideration becomes a half-space. We term the latter *target half-space*. We additionally invoke the Sommerfeld radiation condition [9] in order to assure that no contributions to the desired sound field originate from infinity (where the hemispherical boundary is).

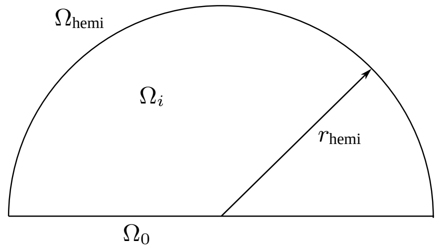


Figure 1: Illustration of a boundary consisting of a disc Ω_0 and a hemisphere Ω_{hemi}.

For convenience, we assume the boundary of our target half-space (i.e. the secondary source distribution) to be located in the x-z-plane, and we assume the target half-space to include the positive y-axis. Refer to Fig. 2. The sound field $S(\mathbf{x}, \omega)$ synthesized by the infinite uniform planar secondary source distribution is then given by [15, 16]

$$S(\mathbf{x}, \omega) = \iint_{-\infty}^{\infty} D(\mathbf{x}_0, \omega) \cdot G(\mathbf{x} - \mathbf{x}_0, \omega)\; dx_0 dz_0 \;, \tag{6}$$

where $\mathbf{x}_0 = [x_0 \; 0 \; z_0]^T$ denotes the position of a secondary source. In order that (6) holds $G(\mathbf{x} - \mathbf{x}_0, \omega)$ has to be invariant with respect to translation along the planar secondary source contour. Integrals like (6) are termed *Fredholm integrals* [14]. Note the resemblance of (6) to the Rayleigh integrals [9].

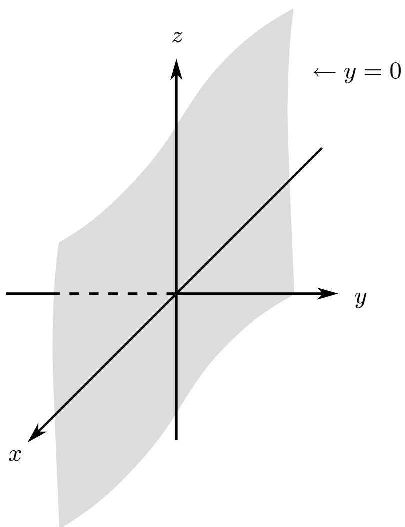


Figure 2: Illustration of the setup of a planar secondary source distribution located along the x-z-plane. The secondary source distribution is indicated by the gray shading and has infinite extent. The target half-space is the half-space bounded by the secondary source distribution and containing the positive y-axis.

As with spherical secondary source contours, (6) can interpreted as a convolution along the secondary source contour. In this case it is a two-dimensional convolution along the spatial dimensions x and z respectively. The according convolution theorem is given by

$$\tilde{S}(k_x, y, k_z, \omega) = \tilde{D}(k_x, k_z, \omega) \cdot \tilde{G}(k_x, y, k_z, \omega) \; , \quad (7)$$

which relates the involved quantities in wavenumber domain [17].

The secondary source driving function is given in wavenumber domain by

$$\tilde{D}(k_x, k_z, \omega) = \frac{\tilde{S}(k_x, y, k_z, \omega)}{\tilde{G}(k_x, y, k_z, \omega)} \; , \quad (8)$$

and in temporal spectrum domain by [15, 16]

$$D(x, z, \omega) = \frac{1}{4\pi^2} \iint_{-\infty}^{\infty} \frac{\tilde{S}(k_x, y, k_z, \omega)}{\tilde{G}(k_x, y, k_z, \omega)} \, e^{-jk_x x} e^{-jk_z z} \, dk_x dk_z \; . \quad (9)$$

In order that $\tilde{D}(k_x, k_z, \omega)$ and $D(x, z, \omega)$ are defined, $\tilde{G}(k_x, y, k_z, \omega)$ may not exhibit zeros. If the latter requirement is not fulfilled in practical situations, regularization can be applied in order to ensure a good behavior of the inverse of $\tilde{G}(\cdot)$. In the presented approach, the regularization can be applied on individual spatial frequencies k_x and k_y respectively which results generally in a more gentle regularization than regularizing the entire inverse problem like in [18].

From (8) and (9) it is obvious that the driving signal is essentially yielded by a division in spatial frequency domain. We therefore refer to the presented approach as *spectral division method* (SDM).

Equation (9) suggests that $D(x, z, \omega)$ is dependent on the distance y of the receiver to the secondary source distribution since y is apparent on the right hand side of (9). It can be shown that under certain circumstances, y does indeed cancel out making $D(x, z, \omega)$ independent from the location of the receiver. Refer to [16] for a further treatment.

4. EXTENSION OF THE AMBISONICS APPROACH TO LINEAR SECONDARY SOURCE DISTRIBUTIONS

A planar secondary source contours like the one treated in Sec. 3 will be rarely implemented in practice due to the enormous amount of loudspeakers necessary. Typically, audio reproduction systems employ linear arrays or a combination thereof and aim at reproduction in the horizontal plane. For convenience, a linear secondary source distribution is assumed which is located along the x-axis (thus $\mathbf{x}_0 = [x_0 \; 0 \; 0]^T$) in the following. Refer to Fig. 3.

For this setup the reproduction equation is given by [15, 16]

$$S(\mathbf{x}, \omega) = \int_{-\infty}^{\infty} D(\mathbf{x}_0, \omega) \cdot G(\mathbf{x} - \mathbf{x}_0, \omega) \, dx_0 \; . \quad (10)$$

Equation (10) can be interpreted as a convolution along the x-axis and the convolution theorem

$$\tilde{S}(k_x, y, z, \omega) = \tilde{D}(k_x, \omega) \cdot \tilde{G}(k_x, y, z, \omega) \quad (11)$$

holds [17]. The secondary source driving function $\tilde{D}(k_x, \omega)$ in

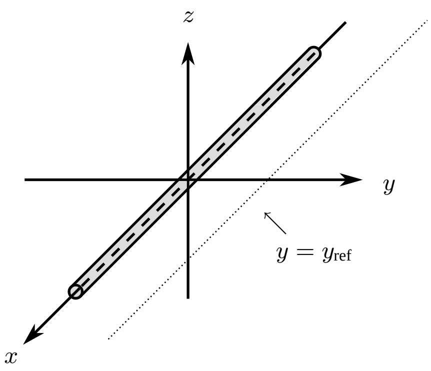


Figure 3: Illustration of the setup of a linear secondary source situated along the x-axis. The secondary source distribution is indicated by the grey shading and has infinite extent. The target half-plane is the half-plane bounded by the secondary source distribution and containing the positive y-axis. The thin dotted line indicates the reference line (see text).

wavenumber domain is thus given by

$$\tilde{D}(k_x, \omega) = \frac{\tilde{S}(k_x, y, z, \omega)}{\tilde{G}(k_x, y, z, \omega)} \; . \quad (12)$$

In the above derivation, we intentionally assumed $D(x, \omega)$ to be exclusively dependent on x because x is the only degree of freedom in the position of the secondary sources. However, generally $D(x, \omega)$ will be dependent on the position of the receiver. This is mathematically reflected by the fact that y and z do not cancel out in (12) [16].

It is not surprising that we are not able to reproduce arbitrary sound fields over an extended area since we are dealing with a secondary source distribution which is neither infinite in two dimensions nor does it enclose the target volume [9].

In the present case, the secondary source setup will only be capable of creating wave fronts that propagate away from it. We will treat this circumstance in an intuitive way in the following. Refer to [16] for a rigorous derivation.

The propagation direction of the reproduced sound field can generally only be correct inside one half-plane bounded by the secondary source distribution. We term this half-plane *target half-plane*. The reproduced sound field anywhere else in space is a byproduct whose properties are determined by the secondary source driving function $D(x, \omega)$ and the radiation characteristics of the secondary sources in the respective direction. For convenience, we aim at reproducing a given desired sound field inside that half of the horizontal plane which contains the positive y-axis. We therefore set $z = 0$.

However, above considerations do not affect the dependence of the driving function on y. In other words, even inside the target half-plane the reproduced sound field will generally only be correct on a line parallel to the x-axis at distance $y = y_{\text{ref}}$ [16]. At locations off this reference line, the reproduced sound field generally deviates from the desired sound field in terms of amplitude, propagation direction, and near-field components [16].

The present situation, i.e. the employment of secondary sources with a three-dimensional spatio-temporal transfer function for two-dimensional reproduction and all resulting properties of the reproduced sound field are termed 2.5-dimensional reproduction, e.g. [2]. This 2.5-dimensional reproduction exhibits special properties as discussed below.

In order to simplify the mathematical treatment, we restrict the validity of equations (10)–(12) to our reference line in the target half-plane, i.e. $z = 0$ and $y = y_{\text{ref}}$.

Equation (12) is then given by

$$\tilde{D}(k_x, \omega) = \frac{\tilde{S}(k_x, y_{\text{ref}}, 0, \omega)}{\tilde{G}(k_x, y_{\text{ref}}, 0, \omega)} \; . \tag{13}$$

Performing an inverse Fourier transform with respect to k_x on (13) yields the driving function $D(x, \omega)$ in temporal spectrum domain as

$$D(x, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\tilde{S}(k_x, y_{\text{ref}}, 0, \omega)}{\tilde{G}(k_x, y_{\text{ref}}, 0, \omega)} \, e^{-jk_x x} \, dk_x \; . \tag{14}$$

In order that $D(x, \omega)$ is defined, $\tilde{G}(k_x, y_{\text{ref}}, 0, \omega)$ may not exhibit zeros. As with planar contours, regularization can be applied in practice in order to ensure a good behavior of the inverse of $\tilde{G}(k_x, y_{\text{ref}}, 0, \omega)$. Refer to [19] for considerations on the incorporation of secondary source directivity.

For purposes of illustration of the basic properties of the presented approach, we treat the scenario of a monochromatic virtual plane wave of frequency ω_{pw} which propagates along the x-y-plane in direction θ_{pw} synthesized by a continuous linear distribution of secondary monopoles. When secondary monopoles are assumed, the inverse of $\tilde{G}(k_x, y_{\text{ref}}, 0, \omega)$ is well defined and no regularization is required.

Considering above described referencing, the secondary source driving function can be shown to be [16]

$$\tilde{D}_{\text{pw}}(k_x, \omega) = \frac{4j \cdot e^{-jk_{\text{pw},y} y_{\text{ref}}}}{H_0^{(2)}(k_{\text{pw},y} y_{\text{ref}})} \cdot 2\pi\delta(k_x - k_{\text{pw},x}) \times \\ \times 2\pi\delta(\omega - \omega_{\text{pw}}) \; , \tag{15}$$

and

$$D_{\text{pw}}(x, \omega) = \frac{4j \cdot e^{-jk_{\text{pw},y} y_{\text{ref}}}}{H_0^{(2)}(k_{\text{pw},y} y_{\text{ref}})} \cdot e^{-jk_{\text{pw},x} x} 2\pi\delta(\omega - \omega_{\text{pw}}) \tag{16}$$

respectively, with $k_{\text{pw},x} = k_{\text{pw}} \cos\theta_{\text{pw}}$ and $k_{\text{pw},y} = k_{\text{pw}} \sin\theta_{\text{pw}}$.

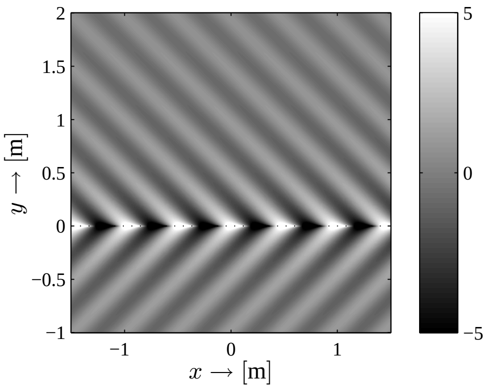
Transferred to the time domain and formulated for broadband signals, (16) reads

$$d_{\text{pw}}(x, t) = \\ f(t) *_t \hat{s}\left(t - \frac{x}{c}\cos\theta_{\text{pw}}\sin\phi_{\text{pw}} - \frac{y_{\text{ref}}}{c}\sin\theta_{\text{pw}}\sin\phi_{\text{pw}}\right) \; . \tag{17}$$

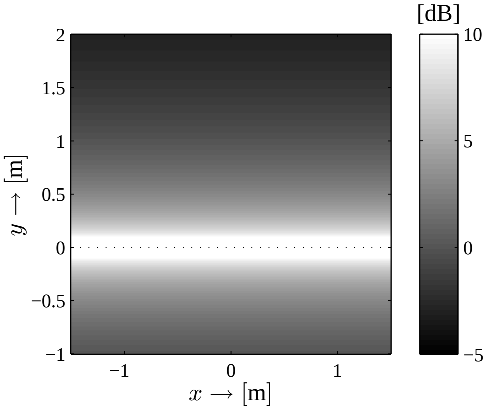
$f(t)$ denotes a filter with frequency response

$$F(\omega) = \frac{4j}{H_0^{(2)}(k_{\text{pw},y} y_{\text{ref}})} \; ,$$

the asterisk $*_t$ denotes convolution with respect to time, and $\hat{s}(t)$ the time domain signal that the plane wave carries. Thus, the time domain driving signal for a secondary source at a given location is yielded by applying a delay and a filter on the time domain input signal. The transfer function $F(\omega)$ of the filter has high pass characteristics with a slope of approximately 3 dB per octave.



(a) $\Re\{S_{\text{pw}}(\mathbf{x}, \omega)\}$



(b) $20 \cdot \log_{10} |S_{\text{pw}}(\mathbf{x}, \omega)|$

Figure 4: Sound pressure $S_{\text{pw}}(\mathbf{x}, \omega)$ of a continuous linear distribution of secondary point sources synthesizing a virtual plane wave of $f_{\text{pw}} = 1000$ Hz and unit amplitude with propagation direction $\theta_{\text{pw}} = \frac{\pi}{4}$ referenced to the distance $y_{\text{ref}} = 1.0$ m. The secondary source distribution is indicated by the dotted line. Only the horizontal plane is shown. The values are clipped as indicated by the colorbars.

$F(\omega)$ is exclusively dependent on the propagation direction of the desired plane wave and on the amplitude reference distance y_{ref}. It is therefore equal for all secondary sources and it is sufficient to perform the filtering only once on the input signal before distributing the signal to the secondary sources. The delay is dependent both on the propagation direction of the desired plane wave and on the position of the secondary source. It therefore has to be performed individually for each secondary source. Note the strong resemblance of this implementation scheme to the implementation of WFS [2, 16].

Fig. 4 depicts simulations of above treated scenario of a virtual plane with propagation direction $\theta_{\text{pw}} = \frac{\pi}{4}$. It can be seen from Fig. 4(a) that the wave fronts are indeed plane. As evident from Fig. 4(b), the synthesized sound field though exhibits an amplitude decay of approximately 3dB for each doubling of the distance to the secondary source distribution. This circumstance is characteristic for 2.5-dimensional reproduction [6].

5. COMPARISON TO WAVE FIELD SYNTHESIS

WFS [1] constitutes the best-known analytical sound field synthesis approach besides Ambisonics. WFS is based on Rayleigh's first integral formula which enables the calculation of the loudspeaker driving signals from the directional gradient of the sound field to be reproduced. In its initial formulation, it assumes that the secondary source distribution is planar in the three-dimensional case and linear in the 2.5-dimensional case which allows a direct comparison to the approach presented in Sec. 3 and 4. Due to the practical relevance we restrict further considerations to linear secondary source distributions.

The close relationship of the presented approach and WFS has already been indicated at the end of Sec. 4. Although the presented approach exhibits similar properties to WFS in practice, the fundamental benefit is the fact that the former provides an exact solution on the reference line. 2.5-dimensional WFS on the other hand, constitutes an approximation even on the reference line [20]. The essential benefit of WFS however is the circumstance that the driving signals can be comfortably calculated via the directional gradient of the virtual sound field which is feasible even for complicated virtual sound fields like moving sources and alike [21] where the SDM-solution has not been found.

The spectral division method has been primarily employed so far as a theoretical tool in order to optimize WFS reproduction. It has been shown that the theoretical superiority of SDM may be exploited to reduce artifacts in WFS while still maintain the practical benefits of the latter. The following enumeration gives an overview over the achievements.

- It has been shown that 2.5-dimensional WFS suffers from systematic amplitude errors additional to those amplitude errors due to the 2.5-dimensionality when virtual plane waves are reproduced [16].
- The derivation of the loudspeaker driving signals for 2.5-dimensional WFS involves two separate stationary phase approximations. As a consequence, the driving signals are only correct when both the virtual source under consideration as well as the receiver are sufficiently far away from the secondary source contour. Especially when the virtual source approaches (or even crosses) the secondary source contour, a systematic coloration is introduced which can be easily overcome using knowledge deduce from the SDM [20].
- The SDM employs a formulation of the reproduction equation in spatial frequency domain, e.g. (11). This representation is very convenient since it allows for a straightforward discrimination of the involved propagating and evanescent sound fields. Such knowledge can be used to optimize the reproduction of focused virtual sound sources in terms of practicability by suppressing all evanescent components in the synthesized sound field [22].

6. CONCLUSIONS

The spectral division method for sound field synthesis was presented. It can be interpreted as an extension of the near-field compensated higher order Ambisonics approach to the employment of planar and linear secondary source contours. The Ambisonics solution which assumes spherical secondary source distributions was interpreted as the single-layer potential solution to the underlying problem. This interpretation directly enables the employment of arbitrarily shaped secondary sources distributions which simply enclose the volume of interest. The solution of the problem is obtained via a formulation of the reproduction equation in a suitable spatial frequency domain. For the spherical secondary source distributions which are typically employed, this suitable spatial frequency domain is given by the spherical harmonics domain.

It was shown that a planar and linear secondary source distributions may also be applied with certain limitations. In this case, a formulation of the reproduction equation in wavenumber domain is used. For all secondary source contours treated in this paper, i.e. spherical, planar, and linear ones, the suitable spatial frequency domain was found as that domain providing a convolution theorem for convolution along the respective secondary source contour.

It was shown that the solution to linear secondary source distributions is exact only on a given reference line. Examples were given how this exact property of the presented approach can be exploited in order to reduce artifacts in wave field synthesis, an alternative yet approximative analytical approach which can also handle linear and planar secondary source distributions.

The operator describing sound field synthesis using planar and linear secondary source distributions is not compact. This fact leads to continuous spatial frequencies. Spherical contours on the other hand constitute a compact operator so that only discrete spatial frequencies (*modes*) are possible.

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