QUIET ZONES FOR ACTIVE CONTROL OF SOUND USING SPHERICAL LOUDSPEAKER ARRAY

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ABSTRACT

Active control of sound can be employed globally to reduce noise levels in an entire enclosure, or locally around a listener's head. Recently, spherical loudspeaker arrays have been studied as multiple-channel sound sources capable of generating sound fields with high complexity. In this paper, several aspects of using a spherical loudspeaker array for local active control of sound are investigated. The first topic under investigation in this paper is the feasibility of creating sphere-shaped quiet zones away from the source. These can be useful in practice to cover the space around a listener's head. However, zones of quiet created away from the source tend to force the secondary source to produce zones of amplification away from the quiet zone. It is shown that by designing quiet zones in the form of a shell around the source, sound amplification can be significantly reduced. Finally, the combination of several spherical sources to enlarge the quiet zone will be demonstrated.

1. INTRODUCTION

Active control of sound has been employed to reduce noise levels around listeners' heads using destructive interference from noise-canceling sound sources [1]. Local active control of sound with a single secondary source typically creates a quiet zone with an extent of a tenth of a wavelength for a reduction of 10 dB in the noise level [2]. An increase in the extent of the quiet zone was shown to be possible when two closely-located secondary sources were used to cancel both pressure and particle velocity [3]. The use of a larger number of secondary sources may produce larger zones of quiet, but active control systems with multiple sources usually require the positioning of multiple loudspeakers in the space around the quiet zone, creating an overall complex and unattractive system.

Recently, spherical loudspeaker sources composed of an array of loudspeaker units mounted around the surface of a sphere, have been used as multiple-channel sources by driving each loudspeaker unit individually [4] [5]. Due to its multiple-channel nature, the spherical loudspeaker array is capable of producing high-order sound fields, in spite of the fact that it is a spatially-compact source. This attractive property of the spherical loudspeaker array may become useful for local active control of sound, facilitating a multiple-channel yet compact system, generating extended quiet zones.

In previous work by Rafaely [6], and Peleg and Rafaely [7], it has been shown that relatively large quiet zones can be generated by the spherical loudspeaker source in various regions near the source, and some ways to limit sound amplifications away from the quiet zone have also been presented. In this paper, the ability to generate a practical 3-D quiet zone is investigated. A spherical shaped quiet zone is investigated first, which is ideal for positioning around a listener's head. Then, shell-shaped quiet zones, which are characterized by very low amplification away from the quiet zone are studied. Finally, the combination of several spherical loudspeaker arrays is investigated, showing the potential for producing large quiet zones without significant amplification.

2. LOCAL ACTIVE CONTROL USING SPHERICAL ARRAY

A spherical loudspeaker array is composed of a set of loudspeaker units mounted around a sphere; by driving each loudspeaker individually, this array becomes a multiple-channel source. The spherical array has been recently studied for local active control [6], and was shown to achieve zones-of-quiet larger than the well known limit of a tenth of wavelength when using a secondary monopole source [2].

First, we will observe the primary sound pressure field, assuming multiple plane waves scattered from a rigid sphere of radius r_0 centered at the origin, representing a spherical source turned off, producing a total sound field (incident and scattered) at radius r written in the spherical harmonics domain as [6]

$$p(k,r,\theta,\phi)=\sum_{n=0}^{\infty}\sum_{m=-n}^{n}4\pi i^n a_{nm}(k)b_n(k,r,r_0)Y_n^m(\theta,\phi) \tag{1}$$

where $Y_n^m(\theta,\phi)$ are the spherical harmonics [8], and $a_{nm}(k)$ is the spherical Fourier transform of the amplitude of the plane waves $a(k,\theta_w,\phi_w)$ propagating towards (θ_w,ϕ_w); $b_n(k,r,r_0)$ has been previously presented [6]. A primary sound pressure field composed from 40 plane waves propagating in various directions is shown in Fig. 1, for a harmonic plane wave at $500Hz$, with $k=9.16$. The secondary sound pressure field produced by the spherical loudspeaker array operating with radial surface velocity u_{nm} is given by [8]:

$$p(k,r,\theta,\phi)=i\rho_0 c\sum_{n=0}^{\infty}\sum_{m=-n}^{n}\frac{h_n(kr)}{h_n'(kr_0)}u_{nm}(k,r_0)Y_n^m(\theta,\phi) \tag{2}$$

such that

$$p_{nm}(k,r)=i\rho_0 c\frac{h_n(kr)}{h_n'(kr_0)}u_{nm}(k,r_0) \tag{3}$$

where $u_{nm}(k, r_0)$ is the spherical Fourier transform of a spherical radial surface velocity, $h_n(\cdot)$ is the spherical Hankel function of the first kind, and $h'_n(\cdot)$ represent derivative. The total sound pressure field is a superposition between the primary sound pressure field (1) and the secondary sound pressure field (2):

$$\begin{aligned} p_{tot}(k, r_q, \theta_q, \phi_q) &= \sum_{n=0}^{\infty} \sum_{m=-n}^{n} 4\pi i^n a_{nm}(k) b_n(k, r_q, r_0) \\ &\times Y_n^m(\theta_q, \phi_q) \\ &+ i\rho_0 c \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \frac{h_n(kr_q)}{h'_n(kr_0)} u_{nm}(k, r_0) \\ &\times Y_n^m(\theta_q, \phi_q) \end{aligned} \quad (4)$$

where (r_q, θ_q, ϕ_q) are the sample points. Equation 4 can be written in a matrix form [6]:

$$\mathbf{p} = \mathbf{s} + \mathbf{A}\mathbf{u} \quad (5)$$

where $\mathbf{p}$ is a $Q \times 1$ vector of the total pressure samples, given by:

$$\mathbf{p} = [p_0, p_1, ..., p_{Q-1}]^T \quad (6)$$

with

$$p_q = p_{tot}(k, r_q, \theta_q, \phi_q) \quad (7)$$

and $\mathbf{s}$ is a $Q \times 1$ vector of the primary pressure sound field samples, but also includes scattering from the source under rigid boundary condition $(u_{nm} = 0)$, given by:

$$\mathbf{s} = [s_0, s_1, ..., s_{Q-1}]^T \quad (8)$$

where each element is given by the first summation in (4)

$$s_q = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} 4\pi i^n a_{nm}(k) b_n(k, r_q, r_0) Y_n^m(\theta_q, \phi_q) \quad (9)$$

The radial surface velocity, u_{nm} , is of order N, and so vector $\mathbf{u}$ is of dimensions $(N+1)^2 \times 1$, given by:

$$\mathbf{u} = [u_{00}, u_{1(-1)}, u_{10}, u_{11}, ..., u_{NN}]^T \quad (10)$$

Matrix $\mathbf{A}$ of dimensions $Q \times (N+1)^2$ is defined by the elements A_{qj} given by:

$$\begin{aligned} A_{qj} &= i\rho_0 c \frac{h_n(kr_q)}{h'_n(kr_0)} Y_n^m(\theta_q, \phi_q) \quad , j = n^2 + n + m, \\ & n \leq N, \; -n \leq m \leq n. \end{aligned} \quad (11)$$

In order to find the radial source velocity that minimizes the 2-norm of the total sound pressure level a least-square solution will be used [9]:

$$\mathbf{u} = -\mathbf{A}^{\dagger}\mathbf{s} \quad (12)$$

where $\mathbf{A}^{\dagger}$ is the pseudo-inverse of $\mathbf{A}$.

3. SPHERE SHAPED QUIET ZONES

Sphere shaped quiet zones designed away from the spherical secondary source are interesting and can be used, for example, to cover a space around a listener‘s head. To ensure zero sound pressure in an enclosure, according to the Kirchoff-Helmholtz theorem, the sound pressure level and the particle normal velocity should be zero on the surface of the enclosure [1]. Since velocity measurements are not commonly used, to simplify the system, only the sound pressure on the surface of the quiet zone sphere will be minimized. To find the source radial velocity that minimizes the sound pressure, a numerical optimization as shown in (12) will be used. Sound attenuation level due to minimization of the sound pressure level on a $0.3m$ sphere centered at $(r, \theta, \phi) = (0.5m, 90^o, 0)$ is shown in Fig. 2.

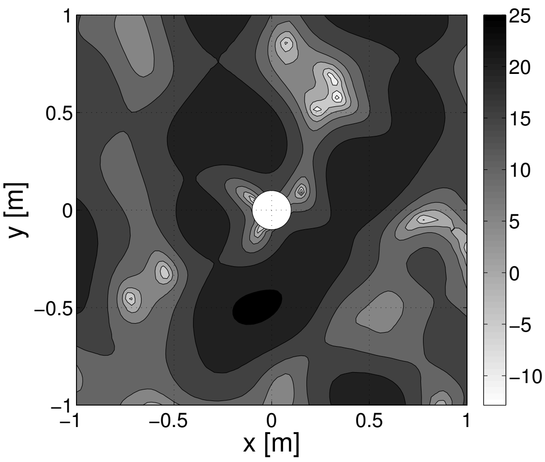

Figure 1: Magnitude in decibels of a multiple plane-waves primary sound field with secondary source switched off.

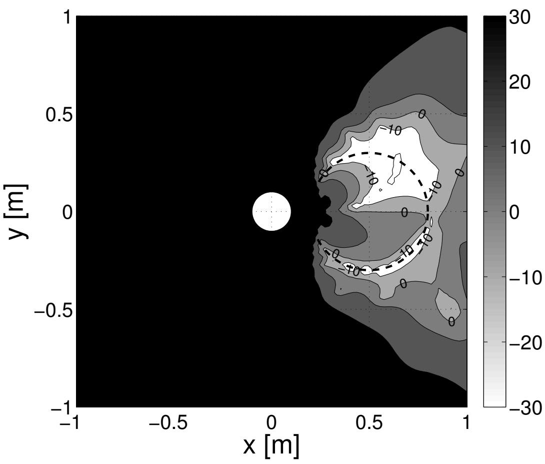

Figure 2: Magnitude in decibels of the sound attenuation with a spherical secondary source designed to minimize the total pressure on a sphere centered at $(r, \theta, \phi) = (0.5m, 90^o, 0)$; the primary sound field is as in Fig. 1.

From Fig. 2 it can be seen that sound pressure minimization on the surface of the quiet zone did not lead to adequate pressure reduction inside the sphere. In order to determine the causes, we will observe the spherical Bessel function that describes the sound pressure level around an open sphere [8]. Since some of the spherical harmonics coefficients can be zero due to the zeros of the spherical Bessel function, two concentric spheres are used in an approach similar to the dual-sphere microphone array [10]. The minimization of sound pressure on two closely located spheres is similar to the minimization of radial surface velocity, and therefore leads to a solution that may satisfy the Kirchoff-Helmholtz conditions. The ratio between the radius of

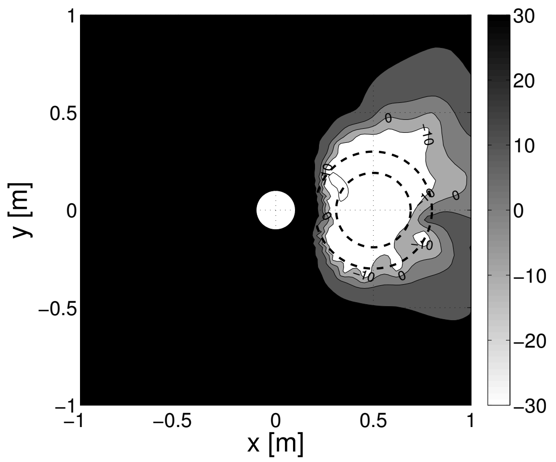

Figure 3: Magnitude in decibels of the sound attenuation with a spherical secondary source designed to minimize the total pressure on a dual sphere centered as in Fig. 2 , the primary sound field as in Fig. 1; source order is $N = 12$.

the outer sphere (r_{out}) and the inner sphere ($r_{internal}$) is defined by [10]:

$$r_{internal} = \frac{r_{out}}{1 + \frac{\pi}{2kr_{out}}} \tag{13}$$

Sound attenuation level due to dual sphere sampling points with an outer sphere as in Fig. 2 and an inner sphere defined by (13) is shown in Fig. 3, presenting an improved sound attenuation.

Another aspect that needs to be determined is the requirement from the spherical loudspeaker array in order to produce such a sound pressure field. The source requirement is defined by the effective spherical-harmonics order of the primary sound pressure field. Therefore, the secondary source order will be equal to or greater than the primary field order. According to (1) the primary sound field depends on $j_n(kr')$ and $h_n(kr')$. Since the effective order of those functions is $N' = kr'$, where k is the wave number and r' is the radius of the spherical quiet zone; the effective spherical-harmonics order of the primary sound field is $N' = kr'$. Now, using the translation theorem, the spherical harmonics decomposition of the primary sound field can be calculated relative to the origin of the secondary spherical source.

According to the theory of translation of spherical harmonics, the coefficient of each spherical harmonic after the translation is a function of all the coefficients existing before the translation [11]. The translation process typically leads to an increase in the effective spherical-harmonics order, depending on the size of the translation and the wavelength. To investigate the effective spherical-harmonics order we focused on the spherical Bessel function $j_n(kr)$. The effective order of the spherical Bessel function is $N \approx kr$, if we assume that r' is the radius of the zone of quiet, r'' is the distance between the center of the spherical source and the center of the sampling sphere, and r is the distance from the source's center and the sampling point on the quiet zone surface, as shown in Fig. 4. Then r is upper bounded by $r' + r''$ and the harmonics of order above $N \approx kr = k(r' + r'')$ can be neglected. This relation shows that the translation caused an increase in the effective spherical-harmonics order of the primary sound field from the secondary source point of view of about kr''; therefore the spherical source order will be $N \approx kr$. Fig. 5 represents the sound attenuation level when the sampling points are as in Fig. 3, but the secondary source is of limited order at $N_{source} = 4 < N = 8$, and the primary sound field is of limited order at $N \approx k(r' + r'') = 9.16(0.3 + 0.5) = 7.33$. The figure shows that indeed results are poorer due to the insufficient order.

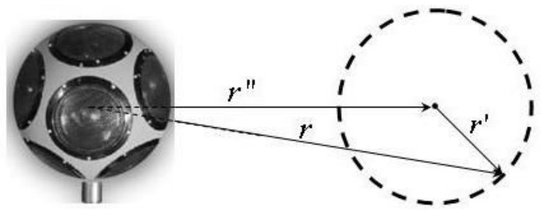

Figure 4: Illustration of coordinate translation.

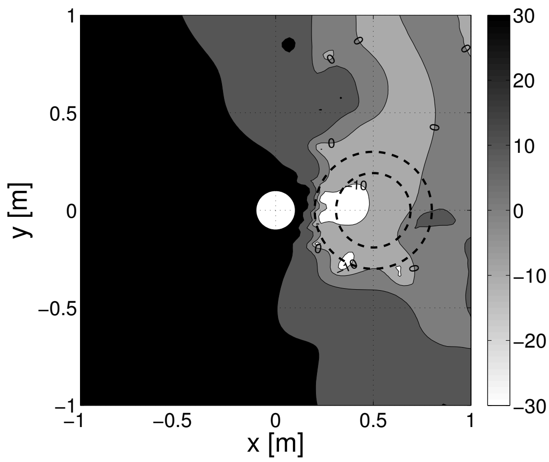

Figure 5: Magnitude in decibels of the sound attenuation with a spherical secondary source designed to minimize the total pressure on a dual sphere centered as in Fig. 3, the primary sound field as in Fig. 1, but source order is reduced to $N = 4$.

However, due to the high amplification of the high harmonics produced by the spherical source, zones of quiet created away from the source tend to force the secondary source to produce zones of amplification away from the quiet zone. It has been previously shown [6] that by designing a more natural quiet zones in the form of a shell around the source, sound amplification can be significantly reduced.

4. SHELL SHAPED QUIET ZONES

The spherical nature of the secondary source led to shell-shaped quiet zones around the source. Contrary to the spherical quiet zone case, where the minimization points are located at a bounded zone and there are no limits on the secondary sound field outside this zone, the design of a shell-shaped quiet zone around the spherical source defines the secondary source in all directions. Therefore the solution did not generate directions with high amplification level. Due to the relation between the source radial velocity and the pressure on the surface of the sphere defined in (2), the choice of shell radius defined the radial source velocity. Sound attenuation levels due to shell-shaped

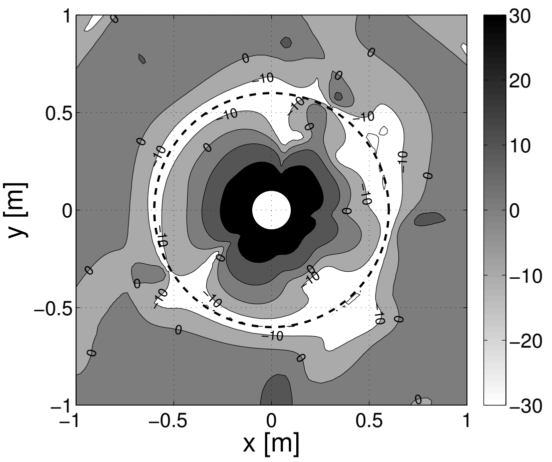

Figure 6: Magnitude in decibels of the sound attenuation with a spherical secondary source designed to minimize the total pressure on a shell shape at $r = 0.6m$ around the source, and the primary sound field as in Fig. 1.

sampling points at radius $r = 0.6m$ are shown in Fig. 6.

There is a reduction in the amplification level far from the spherical source, compared to the spherical quiet zone cases, but close to the spherical source amplification still exists, as can be seen in Fig. 6. In order to investigate this phenomenon, we focused on the relation between the source radial velocity and the sound pressure present in (3). The radius and order dependency is affected by the ratio between the spherical Hankel function and the derivative of the spherical Hankel function. This ratio, presented in Fig. 7, shows that large amplification of u_{nm} is needed to produce high order harmonics; in addition, there is a direct relation between the amplification needed to produce each harmonic and kr. Therefore, an even higher amplification of high orders is required for quiet zones further from the source. Since in order to achieve a quiet zone the effective spherical-harmonics order is $N \approx kr$, increasing the radius of the shell causes an increase of the effective spherical-harmonics order, and according to (3) causes a high amplification level. The amplification needed may lead to ill-conditioning of the solution in (12) or reduced robustness and potential for amplification of electrical noise in practical systems. Fig. 8 represents the sound attenuation level due to shell-shaped sampling points at radius $r = 0.4m$; the amplification level presented is lower than the amplification level present in Fig. 6.

According to this analysis, in order to achieve a well-conditioned quiet zone, the quiet zone should be a non-practical quiet zone located very close to the surface of the spherical loudspeaker source as shown in Fig. 9.

In order to define the conditioning of the solution, the condition number of matrix $\mathbf{A}$ from (12) is calculated. For comparison, Table 1 represents the condition number of matrix $\mathbf{A}$ for each of the above solutions, showing that high condition number is associated with high sound amplification.

5. SEVERAL SPHERICAL SOURCES

The difficulty to produce practical and natural quiet zones led to the use of several spherical sources to control the shape of the quiet zone and simultaneously prevent high amplification regions. The principle is to create a natural shell-shaped quiet zone around each of the spherical sources. At the overlapping area the extent of the quiet zone will increase compared to the extent achieved by a single spherical source. Numerical optimization will be used to find the optimal source velocity, where all sources

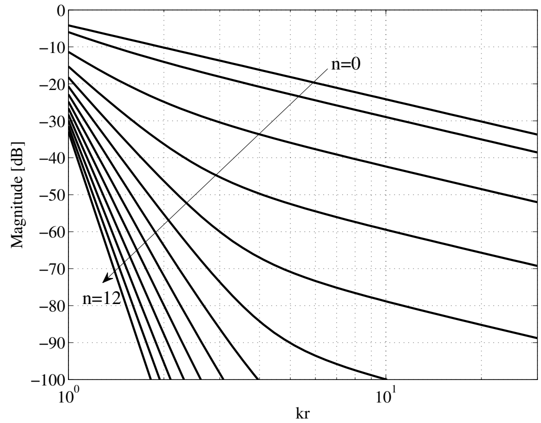

Figure 7: Magnitude of $h_n(kr)/h'_n(kr_0)$ for $n = 0, 1, ..., 12$ as a function of kr, for $kr_0 = 0.916$.

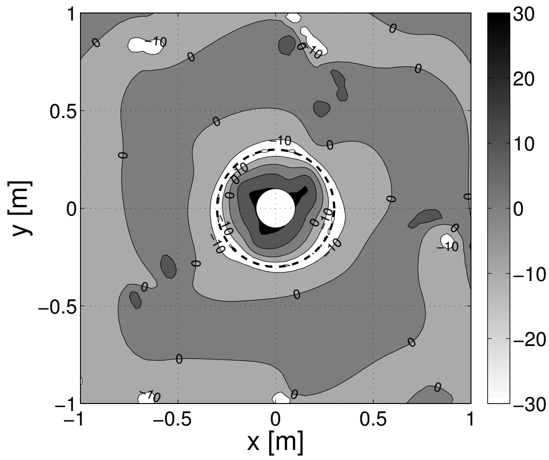

Figure 8: Magnitude in decibels of the sound attenuation with a spherical secondary source designed to minimize the total pressure on a shell shape at $r = 0.3m$ around the source, the primary sound field as in Fig. 1.

Case	N	Fig no.	Cond. no.
Single sphere quiet zone	12	2	1.5×10^{12}
Dual sphere quiet zone	12	3	5.7×10^{11}
Dual sphere quiet zone	4	5	1.1×10^{4}
Shell shaped quiet zone $r = 0.6m$	7	6	5.4×10^{5}
Shell shaped quiet zone $r = 0.3m$	4	8	203
Shell shaped quiet zone $r = 0.12m$	2	9	3.25
Two sources, Shell shaped $r = 0.2m$	4	10	78

Table 1: Comparison between the condition numbers of matrix $\mathbf{A}$ for several cases.

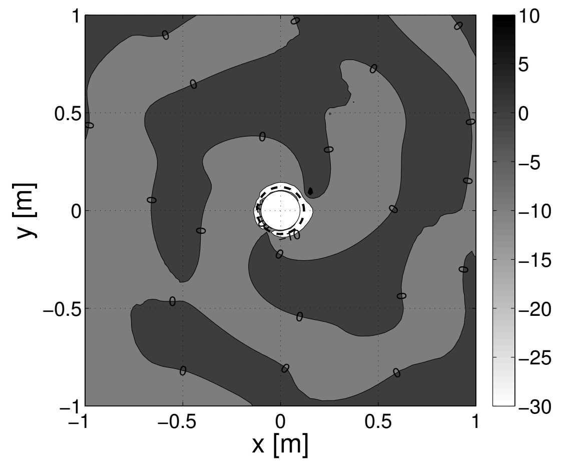

Figure 9: Magnitude in decibels of the sound attenuation with a spherical secondary source designed to minimize the total pressure on a shell shape at $r = 0.12m$ around the source, and the primary sound field as in Fig. 1.

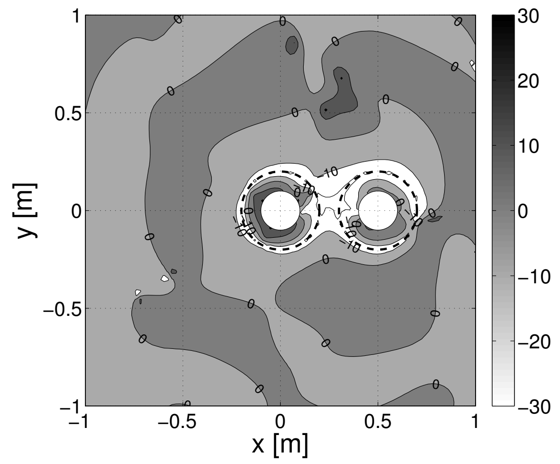

Figure 10: Magnitude in decibels of the sound attenuation with two spherical secondary sources, designed to minimize the total pressure on the shell around each source; the primary sound field as in Fig. 1.

are solved simultaneously. Sound attenuation level was determined as a result of placing two spherical loudspeaker arrays at a distance of $0.5m$, designed to minimize total pressure on the shell-shaped quiet zone around each source, and each shell at a radius of $r = 0.2m$, as shown in Fig. 10. The figure shows that the use of two spherical loudspeaker arrays enlarges the extent of the quiet zone and simultaneously prevents a high amplification level.

6. CONCLUSION

Several aspects of the use of a spherical loudspeaker array for active control of sound have been investigated. The conclusion of the investigation is focused on three aspects, the extent of the quiet zone, the conditions of the solution, and the amplification level away from the quiet zone. Using the spherical quiet zone led to a practical and large quiet zone, but at the expense of unreasonable condition numbers and amplification level. The use of a shell-shaped quiet zone around the spherical source led to a small quiet zone; the solution conditioning and amplification level increase as the radius of the shell increases, but not as high as in the previous case. The use of two spherical sources achieves a reasonable extent of quiet zones and presents a condition number and amplification level similar to the shell shape solution.

7. ACKNOWLEDGEMENT

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