EFFICIENT SAMPLING FOR SCANNING SPHERICAL ARRAY

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ABSTRACT

Spherical microphone arrays have been studied for a wide range of applications. In particular microphones arranged around an open sphere are useful in scanning microphone arrays for sound field analysis. However, open-sphere have been shown to have poor robustness at frequencies related to the zeros of the spherical Bessel functions. This paper presents a new dual ellipsoid shape configuration, which achieves high robustness, with a relatively small number of sampling points. The special geometry of the ellipsoid surface is generated due to a mechanical offset, which allow a full measurement session with one continuous motion of a single microphone.

1. INTRODUCTION

Spherical microphone arrays are widely studied and used for sound field measurement and analysis. The spherical array configuration has an advantage due to its symmetry, which simplifies spatial sound field analysis in three dimensions, and allows efficient spherical harmonics domain processing [1][2][3]. One way to construct a spherical microphone array is to mount microphones on the surface of a rigid sphere [1]. Rigid microphone arrays are limited by size and number of microphones due to practical constraints such as cost, mobility, and the undesired scattering effect of the array on the sound field it measures. Limitations on array size make this method less practical, particularly at low frequencies where arrays of a large extent are required. Limiting the number of microphones leads to limitations on the spatial resolution achievable by processing data measured by the array [3]. Another method to construct a spherical array is around an open sphere. In this method microphones are typically placed on the surface of a virtual sphere in a free field. One way to implement open spherical arrays in applications that do not require real-time or simultaneous recording by all microphones, is by using a single or few microphones making measurement in sequence, in a configuration referred to as a scanning microphone array configuration. Types of such configuration include the dual open sphere array [4] and the shell array [5].

This paper starts with a short review of spherical microphone array processing, and after presenting the advantages and shortcomings of single sphere, dual sphere and spherical shell array configurations, the dual ellipsoid microphone array is presented. This new method combines the advantages of the two previously proposed methods. First, similar to the dual sphere configuration [4], the implementation of the mechanical system based on a single pressure scanning microphone attached to a rotating boom, is relatively simple. Second, the samples in this configuration cover a volume defined by the dual ellipsoid surface, similar to the volume in the spherical shell configuration, so high array robustness is achieved without increasing the number of microphones. Furthermore, due to the special mechanical offset, no adjustment of boom length is required within a measurement session, and the use of a single microphone is sufficient.

2. SPHERICAL MICROPHONE ARRAY PROCESSING

Consider a sound field with pressure denoted by $p\left(k,r,\theta,\phi\right)$, where k is the wave number, and (r,θ,ϕ) is the spatial location in spherical coordinates [3].

Microphone array processing based on spherical harmonics has been recently presented [1][6]. In this section we provide a brief review of this theory, the goal here is finding the plane waves composing the sound field, given the sound pressure measured on the spherical array. We start by describing the pressure on a virtual open sphere. The pressure is due to a unit amplitude incident plane wave, arriving from (θ_l,ϕ_l) and measured at position (r,θ,ϕ), with wave number k. Using spherical harmonics, the pressure can be written as [7]

$$p_l\left(k,r,\theta,\phi\right)=\sum_{n=0}^{\infty}\sum_{m=-n}^{n}b_n\left(kr\right)Y_n^{m^*}\left(\theta_l,\phi_l\right)Y_n^m\left(\theta,\phi\right) \tag{1}$$

where $b_n\left(kr\right)$ is generalized for various array configurations [4] and particularly for single open-sphere pressure microphone arrays $b_n\left(kr\right)=4\pi i^n j_n\left(kr\right)$, where j_n denotes the $n-th$ order spherical Bessel function. $Y_n^m\left(\theta,\phi\right)$ represent the spherical harmonics of order n and degree m. Taking the spherical Fourier transform of $p_l\left(k,r,\theta,\phi\right)$, and using the orthogonality property of spherical harmonics we get

$$p_{l_{nm}}\left(kr\right)=b_n\left(kr\right)Y_n^{m^*}\left(\theta_l,\phi_l\right) \tag{2}$$

For the more general case, where the sound field is composed of an infinite number of plane waves, having spatial amplitude density $a\left(k,\theta,\phi\right)$, p_{nm} can be written as [3]

$$p_{nm}\left(k,r\right)=a_{nm}\left(k\right)b_n\left(k,r\right), \tag{3}$$

where $a_{nm}\left(k\right)$ is the spherical Fourier transform of $a\left(k,\theta,\phi\right)$. Plane wave decomposition can be now performed by dividing the pressure spherical Fourier coefficients by b_n. We wish to avoid dividing by b_n at low values of the spherical Bessel function. We therefore allow microphone positions not to be restricted to a single sphere. This can be done by matrix formulation of the problem. As shown by Rafaely [5], we can find the spherical Fourier transform of the plane wave amplitude density $\mathbf{a_{nm}}$ by

$$\mathbf{a_{nm}} = \mathbf{Cp} \tag{4}$$

where $\mathbf{a_{nm}} = \left[\begin{array}{cccc} a_{00}(k) & a_{1(-1)}(k) & \cdots & a_{NN}(k) \end{array}\right]^{\mathbf{T}}$ and $\mathbf{p} = \left[\begin{array}{ccc} p(k, r_1, \theta_l, \phi_l) & \cdots & p(k, r_M, \theta_M, \phi_M) \end{array}\right]^{\mathbf{T}}$.

Matrix coefficients $\mathbf{C}$ can be calculated using matrix $\mathbf{B}$ defined as:

$$\mathbf{B} = \left[\begin{array}{ccc} b_0(kr_1)\,Y_0^0(\theta_1,\phi_1) & \cdots & b_0(kr_M)\,Y_0^0(\theta_M,\phi_M) \\ b_1(kr_1)\,Y_1^{-1}(\theta_1,\phi_1) & \cdots & b_1(kr_M)\,Y_1^{-1}(\theta_M,\phi_M) \\ b_1(kr_1)\,Y_1^0(\theta_1,\phi_1) & \cdots & b_1(kr_M)\,Y_1^0(\theta_M,\phi_M) \\ b_1(kr_1)\,Y_1^1(\theta_1,\phi_1) & \cdots & b_1(kr_M)\,Y_1^1(\theta_1,\phi_M) \\ \vdots & \ddots & \vdots \\ b_N(kr_1)\,Y_N^N(\theta_1,\phi_1) & \cdots & b_N(kr_M)\,Y_N^N(\theta_M,\phi_M) \end{array}\right]^{\mathbf{T}} \tag{5}$$

and the least squares solution for $\mathbf{C}$ is given by

$$\mathbf{C} = \mathbf{B}^{\dagger} \tag{6}$$

where $\mathbf{B}^{\dagger}$ is the pseudo inverse, $\mathbf{B}^{\dagger} = \left(\mathbf{B^H B}\right)^{-1}\mathbf{B^H}$ assuming $M > (N+1)^2$. Furthermore, an error in the solution is multiplied by the condition number of matrix $\mathbf{B}$. We therefore wish to design a microphone array with a minimal condition number of matrix $\mathbf{B}$, which refers to greater array robustness.

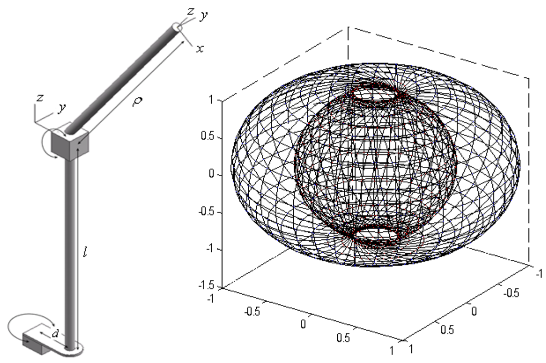
3. PREVIOUSLY INVESTIGATED CONFIGURATIONS

3.1. Scanning microphone array

In this method microphones are typically placed on the surface of a virtual sphere in free field. One way to implement open spherical arrays in applications that do not require real-time or simultaneous recording of all microphones, is by using a single or few microphones making measurement in sequence, in a configuration referred to as a scanning microphone array configuration. The measurements are typically taken by a microphone that is mounted on a boom that is rotated in space using a mechanical apparatus and a set of motors. Different positions of the microphone represent different "microphones" of the microphone array. Unlike the real-time array configuration, the number of spatial samples in this case is not limited, and so a much greater spatial resolution can be achieved compared to real-time array systems, thereby enabling accurate capture of complex sound fields. The length of the boom holding the scanning microphone determines the radius of the open sphere and so analysis of lower frequencies can be performed simply by extending the length of the boom. Scanning microphone array systems are suitable for offline processing applications, such as sound field analysis in auditorium and room acoustics.

3.2. Single open-sphere

A single open-sphere can be very simply implemented as a scanning microphone array configuration. This configuration suffers from ill conditioning at frequencies related to the zeros of the spherical Bessel function. The ill conditioning make this configuration less appropriate for the standard plane wave decomposition analysis.



(a) Mechanical system (b) Dual ellipsoid virtul surface

Figure 1: Scanning microphone mechanical system.

3.3. Dual open-sphere

A dual sphere configuration, obtaining high robustness by combining data from measurements made over two radii is a practical approach for a robust array configuration [4]. However, it comes at the expense of using twice as many microphones (or spatial sampling points) compared to the single sphere configuration. The disadvantage is that the information from microphone samples from only one of the spheres is taken into the calculation for each frequency. Another shortcoming of the dual-sphere configuration is that an adjustment of the second radius has to be made within the measurement session, or two microphones rather than one have to be employed. A method of finding the optimal ratio of the two radii was also presented and will be used in this paper [4].

3.4. Spherical shell

Spherical shell is another configuration where each microphone is placed at a different radius and direction in the volume enclosed by two spheres. The advantage of this method is that it achieves high robustness without increasing the number of microphones, compared to the single-sphere configuration. In the paper by Rafaely [5], the study of different array configurations is realized by comparing the condition number related to each configuration. Array configurations with lower condition numbers are considered more robust to noise and therefore preferable in this respect. The shortcoming of the spherical shell configuration is that the irregular positions in space of the microphones make this configuration less suitable for a scanning microphone array system.
In the next section we present the dual ellipsoid microphone array, which combines the advantages of three previously investigated configurations.

4. DUAL ELLIPSOID MICROPHONE ARRAY

This section presents a scanning microphone array mechanical configuration, based on a single pressure microphone attached to a boom that is mounted on a vertical rod using two circular motors. The mechanical configuration is similar to that of a single sphere scanning microphone array, which is relatively simple to implement. However, in the proposed configuration, a mechanical offset between the two motors generates an off-axis system. This mechanical system allows a constrained motion of the microphone in $\mathbb{R}^3$. By rotating the circular motors a dual ellipsoid virtual surface is generated by microphone positions. As opposed to the dual sphere configuration, where an adjustment of the second radius has to be made within the measurement session, this special geometry allows a full measurement session with one continuous motion of a single microphone. Furthermore, according to the spherical shell configuration, each microphone is located at a different radius and so high robustness can be achieved without increasing the number of microphones compared to a single sphere configuration. A schematic illustration of the mechanical system as well as the geometry of the dual ellipsoid surface is presented in Fig. 1.

4.1. Dual ellipsoid surface geometry

In this section we will describe the geometry of the dual ellipsoid surface with a linear transformation. To represent rotation followed by translation with one linear transformation matrix, we must use homogeneous coordinates [8]. Using homogeneous transformation matrix, translation can be expressed with matrix multiplication. This can be made by increasing the dimension of the transformation matrix by one to absorb the translation part, which is common in projective geometry.

To describe the proposed mechanical system we first define the center of the volume enclosed by the dual ellipsoid surface as the origin of the global coordinate system. Let d be the mechanical offset between the two circular motors, l the length of the vertical rod, and ρ the length of the rotating boom attached to the microphone as shown in Fig. 1 (a). The development of the transformation matrix starts by setting the position of the microphone to $(0, 0, \rho, 1)^T$, where $(0, 0, \rho)^T$ is the position in a local rotating cartesian coordinate system, and the fourth element is the regular addition for homogeneous coordinates. Each rotation of a circular motor followed by translation can be represented as an homogeneous transformation T_i, so the location of the microphone with respect to the global coordinate system can be determined by multiplying two transformation matrices to obtain

$$\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \mathbf{T_2 T_1} \begin{pmatrix} 0 \\ 0 \\ \rho \\ 1 \end{pmatrix} \quad (7)$$

The rotating boom is mounted on the first circular motor, which performs a $pitch$ by ξ (counterclockwise rotation of ξ about the $y-axis$), followed by a translation along the $x-axis$. This translation generates the off-axis system. Another translation is along the $z - axis$ and the total translation is given by $(d, 0, l)$, together with the rotation we get transformation $\mathbf{T_1}$. The second circular motor performs a yaw by ψ (counterclockwise rotation of ψ about the $z - axis$), followed by a second translation along the $z - axis$ given by $(0, 0, -l)$ and together with the rotation can be described by transformation $\mathbf{T_2}$.
The homogeneous transformation matrix that describes the geometry of the ellipsoid surface is therefore given by $\mathbf{T} = \mathbf{T_2 T_1}$

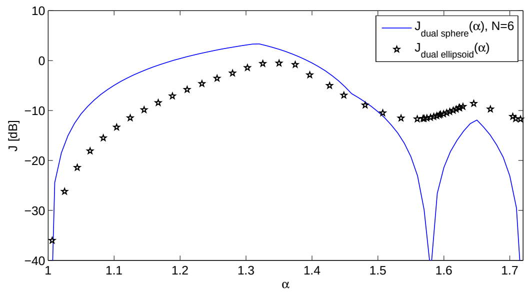


Figure 2: Optimal ratio of ellipsoids radii.

$$\mathbf{T} = \begin{pmatrix} \cos\psi\cos\xi & -\sin\psi & \cos\psi\sin\xi & d\cos\psi \\ \sin\psi\cos\xi & \cos\psi & \sin\psi\sin\xi & d\sin\psi \\ 0 & 0 & \cos\xi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (8)$$

Note that the translations given by $(0, 0, \pm l)$ were made only due to practical reasons and was eventually canceled in the homogeneous transformation $\mathbf{T}$. The position of the microphone is completely determined once d, ρ, ψ, ξ are given. This implies that the mechanically constrained dual ellipsoid has a total of four degrees of freedom. Changing each parameter will change the position of the microphone. ψ controls the azimuth and ξ controls the elevation of the microphone. The angles ψ and ξ act in the same way as ϕ and θ, respectively, in the standard spherical coordinate system [9]. However, as opposed to the ordinary elevation angle $\theta \in (0, \pi)$, ξ is limited only in the range $\xi \in (0, 2\pi)$, which allows the generation of a dual ellipsoid surface, rather than a single ellipsoid or sphere.

Given $\psi \in (0, 2\pi)$ and $\xi \in (0, 2\pi)$, dual ellipsoid surface geometry is completely determined by the ratio of the two mechanical parameters d and ρ. For small d/ρ the two ellipsoids merge and for $d = 0$ we get two centered spheres with equal radii. When d/ρ is increasing from unity a donut shape rather then dual ellipsoid is generated. For a constant ratio d/ρ, extending the length of the rotating boom ρ, will generate a larger array, which will be more desirable for low frequencies.

4.2. Mechanical offset setting

Matrix $\mathbf{B}$ (5) is calculated for different values of k. The zeros of the spherical Bessel function $j_n(kr)$ appear at different values of kr for different orders of n. We first consider a single open sphere array, in which all microphones have equal radii. If for a given $k = k_0, n = n_0$ we get $j_{n_0}(k_0 r) = 0$, then a set of the appropriate rows of $\mathbf{B}(\mathbf{k_0})$ will be zero and the low rank of $\mathbf{B}$ will lead to ill conditioning of the array. The dual ellipsoid spherical shell configuration is motivated by the work of Balmges and Rafaely [4], in which a dual open-sphere configuration was proposed to avoid zeros of $j_n(kr)$. The optimal ratio of the two radii was derived, and so a zero of $j_n(kr)$ for one radius means that the second radius with the same kr and n will not be close to zero. In the dual ellipsoid configuration, microphones are distributed over two virtual ellipsoid surfaces. The distance from each microphone position to the origin refers to the radius r. Microphone positions with different elevation angles have different radii. Radii of sampling points over the

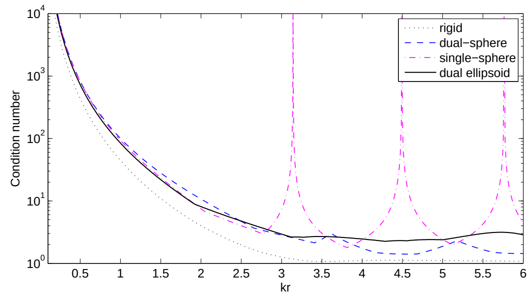


Figure 3: Condition of matrix $\mathbf{B}$, 6-th order array.

external ellipsoid are in the range $\sqrt{d^2+\rho^2} \leq r_{out} \leq d+\rho$, and the radii of sampling points over the internal ellipsoid are in the range $\rho - d \leq r_{in} \leq \sqrt{d^2+\rho^2}$. Let α_{ell} be the ratio between the mean radius of external and internal ellipsoid $\alpha_{ell.} = \bar{r}_{out}/\bar{r}_{in}$, where $\bar{r}$ is the mean value of ellipsoid radius. We wish to find the optimal ratio that will provide the lowest condition number of matrix $\mathbf{B}$ and highest robustness of the microphone array. Since a rigid sphere array configuration does not suffer from ill conditioning, its condition number can be considered as a lower bound for other open-sphere array condition numbers. We define a criterion $J(\alpha_{ell.})$ to examine value of $\alpha_{ell.}$ for optimal robustness of the array configuration.

$$\alpha_{opt} = \arg\max_{\alpha_{ell.}} J(\alpha_{ell.}) \tag{9}$$

$$J(\alpha_{ell.}) = \min_{k} \left\{ \frac{cond\left(\mathbf{B_{rigid}}\left(\mathbf{k}\right)\right)}{cond\left(\mathbf{B_{ell.}}\left(\mathbf{k}\right)\right)} \right\} \tag{10}$$

This criterion is different from the criterion for optimal ratio value found for the dual sphere [4]. Similar to the criterion presented in [4], a greater value of $J(\alpha_{ell.})$ will refer to a better array configuration. The magnitude of $J(\alpha_{ell.})$ is computed numerically for different radii ratios, and is presented in Fig. 2. It is clear from the figure that even though the criteria were different, the optimal value α_{opt} is similar for both methods.

5. SPATIAL SAMPLING

In the previous section the geometry of the dual ellipsoid surface was characterized. To design a microphone array configuration with high robustness to noise, the sampling scheme has to be defined. Several sampling schemes were previously presented [6], that offer a tradeoff between the required number of microphones for implementation on Nth order array, and the simplicity of their arrangement.

5.1. Nearly uniform sampling

Uniform sampling is a scheme where neighboring microphones are distributed at a constant distance called 'Platonic Solids'. For a nearly uniform scheme the distance from neighbor microphones is not constant, but the requirement for exact sampling takes place with equal weights [6]. The advantage of this scheme is the small number of sampling points. Around $1.5\,(N+1)^2$ microphones are required for an $N'th$ order array.

5.2. Gaussian sampling

The Gaussian sampling requires $2\,(N+1)^2$ microphones, which is slightly higher than in a nearly uniform scheme, however, the arrangement of this scheme is much simpler. The azimuth ϕ is sampled within $2\,(N+1)$ equally spaced angles that can be considered as stripes. The elevation angle θ requires only $(N+1)$ nearly equally spaced angles. Larger spacing between θ angles close to the sphere poles reduces sample density at the poles.

5.3. Dual ellipsoid spatial sampling

The standard schemes refer to samples over a single sphere surface, whereas here samples can be placed over a dual ellipsoid surface. Therefore, only the directions of the microphones are taken into account and not their radii. We need to determine whether each microphone position is projected on the external or internal ellipsoid surface. Since the implementation of a dual ellipsoid array is based on a scanning microphone array, the simplicity of the arrangement is not considered an advantage for mechanical reasons but for the simplicity of defining the projection rule.
The sampling scheme is important for array robustness. We wish to utilize the volume enclosed by the ellipsoid surface and so get maximal dispersion [8]. Maximal dispersion means we leave minimal uncovered areas between neighboring microphones ,which lead to a lower condition number of the matrix $\mathbf{B}$. The complexity of matrix $\mathbf{B}$ makes it very difficult to find an analytical solution for distributing the microphones, therefore, numerical solutions are proposed.

The first proposed scheme is based on standard Gaussian sampling. Directions of microphone positions are determined by the directions of the Gaussian sampling scheme. The simplicity of the Gaussian scheme allows definition of a simple rule for determining which microphone will be placed on which ellipsoid. Microphones are alternately projected onto external and internal ellipsoids, so the four closest neighbors of each microphone have different projections from the centered microphone. This rule leads to maximal dispersion and so to maximal robustness compared with other array designs based on Gaussian sampling. Fig. 3. represents the condition number of matrix $\mathbf{B}$ over the range of kr, for different sampling schemes. The proposed scheme overcomes the ill conditioning problem of the single sphere. Furthermore this scheme achieves high robustness, with condition number larger by less than 1dB relative to the dual sphere array configuration, with smaller number of the microphones required for the dual sphere configuration.

Better robustness with a reduced number of microphones may be achieved by using global search methods such as Genetic Algorithms (GA) based on nearly uniform design. Since computation of large matrix condition numbers has to be calculated for various frequencies for each iteration these numeric solutions take a significant time to converge and are suitable only for a specific array order. Furthermore preliminary study showed that the improvement of the overall condition number is limited. It is therefore most convenient to use the proposed Gaussian based scheme, which achieves high robustness with a relatively small number of microphones, and offers a very flexible and simple way to implement scanning pressure microphone arrays.

6. CONCLUSION

This paper presented a new spherical array configuration. This configuration of dual ellipsoid shape achieves high array robustness, with a relatively small number of sampling points. Methods for design and implementation of such an array for different orders or size are given, making the array usable in practice for various applications.

7. ACKNOWLEDGMENT

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